

TeV scale mirage mediation in NMSSM

Tatsuo Kobayashi¹, Hiroki Makino², Ken-ichi Okumura²,
Takashi Shimomura³, and Tsubasa Takahashi⁴

¹*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

²*Department of Physics, Kyushu University, Fukuoka 812-8581, Japan*

³*Department of Physics, Niigata University, Niigata 950-2181, Japan*

⁴*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

Abstract

We study the next-to-minimal supersymmetric standard model. We consider soft supersymmetry breaking parameters, which are induced by the mirage mediation mechanism of supersymmetry breaking. We concentrate on the mirage mediation, where the so-called mirage scale is the TeV scale. In this scenario, we can realize the up-type Higgs soft mass of $\mathcal{O}(200)$ GeV, while other masses such as gaugino masses and stop masses are heavy such as 1 TeV or more. Cancellation between the effective μ -term and the down-type Higgs soft mass ameliorates the fine-tuning in the electroweak symmetry breaking even for $\mu = \mathcal{O}(500)$ GeV. The mixing between the doublet and singlet Higgs bosons is suppressed by $(\lambda/\kappa)\tan^{-1}\beta$. Then the lightest doublet Higgs mass naturally reaches 125 GeV lifted by the new quartic coupling. The higgsino and singlino are light and their linear combination is the lightest superparticle.

1 Introduction

Supersymmetric extension is a good candidate for physics beyond the standard model (SM). The minimal supersymmetric standard model (MSSM) is the simplest extension. The MSSM is quite interesting because of its minimality and its detailed studies have been done for several aspects.

However, the MSSM has the fine-tuning problem. Within the framework of the MSSM, the Z -boson mass, m_Z , is obtained as

$$\frac{m_Z^2}{2} \approx -m_{H_u}^2 - |\mu|^2, \quad (1.1)$$

where $m_{H_u}^2$ is the soft supersymmetry (SUSY) breaking scalar mass squared of the up-sector Higgs field and μ is the supersymmetric mass. The radiative corrections on $m_{H_u}^2$ are obtained as $m_{H_u}^2 \sim -m_{\tilde{t}}^2 \sim -M_3^2$, where $m_{\tilde{t}}$ and M_3 denote the stop and gluino masses, respectively. In most of cases, the stop and the gluino masses are much larger than m_Z . Thus, we need fine-tuning between $m_{H_u}^2$ and $|\mu|^2$ to realize the correct value of m_Z . Furthermore, it is required that $m_{\tilde{t}} = \mathcal{O}(1)$ TeV or larger in order to obtain the Higgs mass such as $m_h \approx 125$ GeV which is recently reported by ATLAS and CMS collaborations[2, 3].

The mirage mediation is one of the interesting mediation mechanisms of SUSY breaking [4, 5, 6]. The mirage mediation is a mixture of the modulus mediation [7] and the anomaly mediation [8] with a certain ratio. In particular, it was pointed out that the TeV-scale mirage mediation can ameliorate the above fine-tuning problem of the MSSM [9, 10, 11]. In the TeV-scale mirage mediation, the above radiative corrections on $m_{H_u}^2$ and the anomaly mediation contributions are canceled each other. Then, the value of $|m_{H_u}^2|$ at the electroweak scale can be smaller than stop and gluino masses. The TeV scale mirage mediation also leads several phenomenologically interesting aspects [12] because its SUSY particle spectrum is quite compressed.

The next-to-minimal supersymmetric standard model (NMSSM) is the extension of the MSSM by adding a singlet S [13] (see for review e.g. [14]). Here, we also impose the Z_3 symmetry. The NMSSM does not have the μ -term, $\mu H_u H_d$, in the superpotential, where H_u and H_d denote the up and the down-sector Higgs superfields, respectively. On the other hand, the term $\lambda S H_u H_d$ is allowed in the NMSSM superpotential. After S develops its vacuum expectation value (VEV), the effective μ -term is generated. That gives us a solution for the so-called μ -problem [15]. The NMSSM is also interesting in the light of the recent indication of the relatively heavy Higgs boson reported by ATLAS and CMS, endowed with an additional Higgs self-coupling. In the Higgs sector, the doublet Higgs and singlet fields mix each other. Then, the Higgs sector in the NMSSM has a quite rich structure.

The NMSSM also leads the same relation as (1.1), and the NMSSM has the fine-tuning problem similar to the one in the MSSM. Thus, it is interesting to apply the TeV-scale mirage mediation to the NMSSM. In this paper, we study the NMSSM with the soft SUSY breaking terms induced through the TeV-scale mirage mediation [1].

This paper is organized as follows. In section 2, we give a brief review on the mirage mediation, and the TeV scale mirage mediation. In section 3, we apply the TeV scale mirage mediation to the NMSSM, and study its spectrum. Section 4 is devoted to conclusion and discussion. In Appendix A, we show explicitly initial conditions of soft parameters, which are induced through the mirage mediation in the NMSSM.

2 TeV-scale mirage mediation

Here, we review briefly the mirage mediation [4]. The mirage mediation is the mixture between the modulus mediation and the anomaly mediation. Then, the gaugino masses are obtained as

$$M_a = M_0 + \frac{m_{3/2}}{8\pi^2} b_a g_a^2, \quad (2.1)$$

where g_a and b_a are the gauge couplings and their β function coefficients, and $m_{3/2}$ denotes the gravitino mass. The first and the second terms in the right-hand side of Eq. (2.1) correspond to the contributions due to the modulus mediation and the anomaly mediation, respectively.

Similarly, we obtain the so-called A -terms corresponding to the Yukawa couplings, y_{ijk} , and the soft scalar masses m_i as

$$\begin{aligned} A_{ijk}(M_{GUT}) &= a_{ijk} M_0 - (\gamma_i + \gamma_j + \gamma_k) \frac{m_{3/2}}{8\pi^2}, \\ m_i^2(M_{GUT}) &= c_i M_0^2 - \dot{\gamma}_i \left(\frac{m_{3/2}}{8\pi^2} \right)^2 - \frac{m_{3/2}}{8\pi^2} M_0 \theta_i, \end{aligned} \quad (2.2)$$

where

$$\begin{aligned} \gamma_i &= 2 \sum_a g_a^2 C_2^a(\phi^i) - \frac{1}{2} \sum_{jk} |y_{ijk}|^2, \\ \theta_i &= 4 \sum_a g_a^2 C_2^a(\phi^i) - \sum_{jk} a_{ijk} |y_{ijk}|^2, \\ \dot{\gamma}_i &= 8\pi^2 \frac{d\gamma_i}{d \ln \mu_R}. \end{aligned} \quad (2.3)$$

Here, $C_2^a(\phi^i)$ denotes the quadratic Casimir corresponding to the representation of the matter field ϕ^i . In addition, a_{ijk} and c_i parametrize the A -term and the scalar mass squared generated through the modulus mediation in the unit of the universal gaugino mass, M_0 . These coefficients are determined by modulus-dependence of the Kähler metric as well as the Yukawa coupling. One can write c_i as

$$c_i = c_i^{(\text{tree})} + \delta c_i^{(\text{loop})}. \quad (2.4)$$

Here, $c_i^{(\text{tree})}$ is calculated from the tree-level Kähler metric of the matter field ϕ^i and they are ratios of small integers including 0 and 1 [7, 16, 11]. In addition, $\delta c_i^{(\text{loop})}$ is obtained

with the one-loop Kähler metric of the matter field, but such a loop correction to the Kähler metric depends on the detail of the ultraviolet-model and is hard to calculate (see e.g. Ref. [17]). That is, c_i is ambiguous at the one-loop level, although such ambiguity is subdominant and less important in most of cases. Here, we consider the case with

$$a_{ijk} = c_i + c_j + c_k. \quad (2.5)$$

We input the values of c_i at $M_{GUT} = 2 \times 10^{16}$ GeV.

It is convenient to use the following parameter [5],

$$\alpha \equiv \frac{m_{3/2}}{M_0 \ln(M_{pl}/m_{3/2})}, \quad (2.6)$$

to represent the ratio of the anomaly mediation to the modulus mediation. Here M_{pl} is the reduced Planck scale.

One of the interesting aspects in the mirage mediation is that the above spectrum (2.1) and (2.2) has a special energy scale, that is, the mirage scale,

$$M_{\text{mir}} = \frac{M_{GUT}}{(M_{pl}/m_{3/2})^{\alpha/2}}. \quad (2.7)$$

At this scale, the gaugino masses are obtained as [5],

$$M_a(M_{\text{mir}}) = M_0. \quad (2.8)$$

That is, the anomaly mediation contribution and the radiative corrections cancel each other, and the pure modulus mediation appears at the mirage scale. Furthermore, the A -terms and the scalar masses squared also satisfy¹

$$A_{ijk}(M_{\text{mir}}) = (c_i + c_j + c_k)M_0, \quad m_i^2(M_{\text{mir}}) = c_i M_0^2, \quad (2.9)$$

if the corresponding Yukawa couplings are small enough or if the following conditions are satisfied,

$$a_{ijk} = c_i + c_j + c_k = 1, \quad (2.10)$$

for non-vanishing Yukawa couplings, y_{ijk} [5].

When $\alpha = 2$, the mirage scale M_{mir} is around 1 TeV. Then, the above spectrum (2.8) and (2.9) is obtained at the TeV scale. That is the TeV scale mirage mediation scenario. In particular, there would appear a large gap between M_0 and the scalar mass m_i with $c_i \approx 0$. We will apply the TeV scale mirage scenario to the NMSSM in the next section.

In the TeV scale mirage scenario, the stop mass squared becomes negative at high energy [18], while it is positive at low energy below 10^6 GeV. Thus, the vacuum which

¹The scalar masses at the Mirage scale can be modified due to the U(1) tadpole contribution in the renormalization group running when the different values of c_{H_u} and c_{H_d} are chosen. However, such a modification is small and can be included in ambiguities of c_i if couplings are small. See [5] for detailed discussions. We include this contribution in our numerical analysis.

breaks the electroweak symmetry at the electroweak scale might be a local minimum, but instead there would be a color and/or charge breaking vacuum with field values larger than 10^6 GeV. Here, we assume the thermal history of the Universe such that field values remain around the origin until the temperature reaches the electroweak scale. In addition, we need to confirm that the tunnelling rate is small enough, i.e. less than the Hubble expansion rate. In Refs. [19], it has been shown that such a rate is small enough, as long as the squark/slepton masses squared are vanishing or positive around 10^4 GeV. This condition is satisfied in our TeV scale mirage mediation scenario.

3 TeV scale mirage in NMSSM

In this section, we apply the TeV scale mirage mediation scenario to the NMSSM.

3.1 NMSSM

Here, we briefly review on the NMSSM, in particular its Higgs sector before we apply the TeV scale mirage mediation scenario to the NMSSM. In the NMSSM, we extend the MSSM by adding a singlet chiral multiplet S and imposing a Z_3 symmetry. Then, the superpotential of the Higgs sector is written as

$$W_{\text{Higgs}} = -\lambda S H_u H_d + \frac{\kappa}{3} S^3. \quad (3.1)$$

Here and hereafter, for S , H_u and H_d we use the convention that the superfield and its lowest component are denoted by the same letter. The full superpotential also includes the Yukawa coupling terms between the matter fields and the Higgs fields, which are the same as those in the MSSM.

The following soft SUSY breaking terms are induced in the Higgs sector,

$$V_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 - \lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 + h.c. \quad (3.2)$$

Then, the scalar potential of the neutral Higgs fields is given as

$$\begin{aligned} V = & \lambda^2 |S|^2 (|H_d^0|^2 + |H_u^0|^2) + |\kappa S^2 - \lambda H_d^0 H_u^0|^2 + V_D \\ & + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 - \lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 + h.c., \end{aligned} \quad (3.3)$$

with

$$V_D = \frac{1}{8} (g_1^2 + g_2^2) (|H_d^0|^2 - |H_u^0|^2)^2, \quad (3.4)$$

where g_1 and g_2 denote the gauge couplings of $U(1)_Y$ and $SU(2)$.

The minimum of the potential is obtained by analyzing the stationary conditions of the Higgs potential,

$$\begin{aligned}\frac{\partial V}{\partial H_d^0} &= \lambda^2 v \cos \beta (s^2 + v^2 \sin^2 \beta) - \lambda \kappa v s^2 \sin \beta + \frac{1}{4} g^2 v^3 \cos \beta \cos 2\beta \\ &\quad + m_{H_d}^2 v \cos \beta - \lambda A_\lambda v s \sin \beta = 0,\end{aligned}\tag{3.5a}$$

$$\begin{aligned}\frac{\partial V}{\partial H_u^0} &= \lambda^2 v \sin \beta (s^2 + v^2 \cos^2 \beta) - \lambda \kappa v s^2 \cos \beta - \frac{1}{4} g^2 v^3 \sin \beta \cos 2\beta \\ &\quad + m_{H_u}^2 v \sin \beta - \lambda A_\lambda v s \cos \beta = 0,\end{aligned}\tag{3.5b}$$

$$\frac{\partial V}{\partial S} = \lambda^2 s v^2 + 2\kappa^2 s^3 - \lambda \kappa v^2 s \sin 2\beta + m_S^2 s - \frac{1}{2} \lambda A_\lambda v^2 \sin 2\beta + \kappa A_\kappa s^2 = 0,\tag{3.5c}$$

where $g^2 = g_1^2 + g_2^2$. Here, we denote VEVs as

$$v^2 = \langle |H_d^0|^2 \rangle + \langle |H_u^0|^2 \rangle, \quad \tan \beta = \frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle}, \quad s = \langle S \rangle.\tag{3.6}$$

Using the above stationary conditions, we obtain the Z boson mass $m_Z^2 = \frac{1}{2} g^2 v^2$ as

$$m_Z^2 = \frac{1 - \cos 2\beta}{\cos 2\beta} m_{H_u}^2 - \frac{1 + \cos 2\beta}{\cos 2\beta} m_{H_d}^2 - 2\mu^2,\tag{3.7}$$

where $\mu = \lambda s$. For $\tan \beta \gg 1$, this equation becomes

$$m_Z^2 \simeq -2m_{H_u}^2 + \frac{2}{\tan^2 \beta} m_{H_d}^2 - 2\mu^2.\tag{3.8}$$

This relation is the same as the one in the MSSM. Indeed, when we neglect the second term in the right-hand side, the above relation is nothing but Eq. (1.1). Thus, the natural values of $|m_{H_u}|$ and $|\mu|$ would be of $\mathcal{O}(100)$ GeV. Furthermore, the natural value of $|m_{H_d}|/\tan \beta$ would be of $\mathcal{O}(100)$ GeV or smaller. Alternatively, $|\mu|$ and $|m_{H_d}|/\tan \beta$ could be larger than $\mathcal{O}(100)$ GeV when μ^2 and $m_{H_d}^2/\tan^2 \beta$ are canceled each other in the above relation at a certain level. Even in such a case, $|m_{H_u}|$ would be naturally of $\mathcal{O}(100)$ GeV. On the other hand, other sfermion masses as well as gaugino masses must be heavy as the recent LHC results suggested. To realize such a spectrum, we apply the TeV scale mirage mediation in the next section, where we take $c_{H_u} = 0$ to realize a suppressed value of $|m_{H_u}|$ compared with M_0 .

3.2 TeV scale mirage mediation in NMSSM

Here, we study the TeV scale mirage mediation scenario in the NMSSM. Soft SUSY breaking terms are obtained through the generic formulas (2.1) and (2.2) with taking $\alpha = 2$. For concreteness, we give explicit results of all the soft SUSY breaking terms for the NMSSM in Appendix A. We concentrate on the Higgs sector as well as gauginos and stops.

We consider the following values of c_i ,

$$c_{H_d}^{(\text{tree})} = 1, \quad c_{H_u}^{(\text{tree})} = 0, \quad c_S^{(\text{tree})} = 0, \quad c_{t_L}^{(\text{tree})} = c_{t_R}^{(\text{tree})} = \frac{1}{2}, \quad (3.9)$$

for H_d , H_u , S , and left and right-handed (s)top fields, respectively. This is the same assignment as the pattern II in Ref. [11] for the MSSM except for c_S . Then, the soft parameters due to only modulus mediation contribution are given by

$$\begin{aligned} (A_t)_{\text{modulus}} &= (A_\lambda)_{\text{modulus}} = M_0, & (A_\kappa)_{\text{modulus}} &= 0, \\ (m_{H_d}^2)_{\text{modulus}} &= M_0^2, & (m_{t_L}^2)_{\text{modulus}} &= (m_{t_R}^2)_{\text{modulus}} = \frac{1}{2}M_0^2, \\ (m_{H_u}^2)_{\text{modulus}} &= (m_S^2)_{\text{modulus}} = 0, \end{aligned} \quad (3.10)$$

when we neglect $\delta c_i^{(\text{loop})}$. The above assignment of c_i (3.9) satisfies the condition, (2.10) for the top Yukawa coupling and the coupling λ , but not for the coupling κ . However, we do not consider a large value of κ to avoid the blow-up of κ and λ as will be shown later. Thus, we obtain the following values,

$$\begin{aligned} A_t &\approx A_\lambda \approx M_0, \\ m_{H_d}^2 &\approx M_0^2, \quad m_{t_L}^2 \approx m_{t_R}^2 \approx \frac{1}{2}M_0^2, \end{aligned} \quad (3.11)$$

up to $\mathcal{O}(\kappa^2/8\pi^2)$ at the TeV scale. Note that $\delta c_i^{(\text{loop})}$ has negligible effects for these values.

Similarly, we can obtain the values of A_κ , $|m_{H_u}|$ and $|m_s|$ at the TeV scale, however those are suppressed compared with M_0 . For such suppressed values, sub-leading corrections e.g. the one-loop correction on the Kähler metric are not negligible anymore. That introduces one-loop ambiguity into the model. Including such corrections, at the TeV scale we obtain

$$m_{H_u}^2 \approx \delta c_{H_u}^{(\text{loop})} M_0^2, \quad m_S^2 \approx \delta c_S^{(\text{loop})} M_0^2, \quad (3.12)$$

with $\delta c_{H_u}^{(\text{loop})}, \delta c_S^{(\text{loop})} = \mathcal{O}(1/8\pi^2)$. Note that similar to Eq.(3.11), Eq.(3.12) also includes corrections of $\mathcal{O}(\kappa^2 M_0^2/8\pi^2)$ due to the violation of the mirage unification by κ . That is, we obtain $m_{H_u}^2 = 0, m_S^2 = 0$ up to $\mathcal{O}(M_0^2/8\pi^2)$ at the TeV scale. Similarly, at the TeV scale we can obtain,

$$A_\kappa = 0, \quad (3.13)$$

up to $\mathcal{O}(M_0/8\pi^2)$. Because of such ambiguity, we use A_κ as a free parameter, which must be small compared with M_0 . In addition, we determine the values of $m_{H_u}^2$, m_S^2 and $\mu (= \lambda s)$ at the electroweak scale from the stationary conditions, (3.5), where we use the experimental value $m_Z = \frac{1}{\sqrt{2}}gv = 91.19$ GeV and $\tan \beta$ as a free parameter.

Through the above procedure, the parameters, $m_{H_u}^2$, m_S^2 and μ , at the electroweak

scale are expressed by $\tan \beta$, $m_{H_d}^2$, A_λ as follows,

$$\begin{aligned}\mu &= \lambda \langle S \rangle = \frac{A_\lambda \tan \beta}{2 \left(1 - \frac{\kappa}{\lambda} \tan \beta\right)} \left\{1 - \sqrt{1 - 4X}\right\}, \\ m_S^2 &= -2 \left(\frac{\kappa}{\lambda}\right)^2 \mu^2 - \left(\frac{\kappa}{\lambda}\right) A_\kappa \mu + \frac{\lambda^2}{g^2} m_Z^2 \left\{ \left(\frac{A_\lambda}{\mu} + 2\frac{\kappa}{\lambda}\right) \sin 2\beta - 2 \right\}, \\ m_{H_u}^2 &= \frac{\tan^2 \beta - 1}{\tan^2 \beta} \left(\frac{m_{H_d}^2}{\tan^2 \beta - 1} - \mu^2 - \frac{m_Z^2}{2} \right),\end{aligned}\quad (3.14)$$

where,

$$X = \frac{m_{H_d}^2 (1 - \frac{\kappa}{\lambda} \tan \beta)}{A_\lambda^2 \tan^2 \beta} \left\{ 1 + \frac{\tan^2 \beta}{\tan^2 \beta + 1} \left(\frac{2\lambda^2}{g^2} - \frac{\tan^2 \beta - 1}{2 \tan^2 \beta} \right) \frac{m_Z^2}{m_{H_d}^2} \right\}. \quad (3.15)$$

For $\tan \beta \gg \max(1, \kappa/\lambda)$, these parameters are approximated as,

$$\mu = \lambda \langle S \rangle \sim \frac{m_{H_d}^2}{A_\lambda \tan \beta}, \quad (3.16a)$$

$$m_S^2 \sim -2 \left(\frac{\kappa}{\lambda}\right)^2 \left(\frac{m_{H_d}^2}{A_\lambda \tan \beta} \right)^2 - \left(\frac{\kappa}{\lambda}\right) A_\kappa \left(\frac{m_{H_d}^2}{A_\lambda \tan \beta} \right) + 2 \frac{\lambda^2}{g^2} \frac{A_\lambda^2}{m_{H_d}^2} m_Z^2, \quad (3.16b)$$

$$m_{H_u}^2 \sim \frac{m_{H_d}^2}{\tan^2 \beta} - \frac{m_{H_d}^4}{A_\lambda^2 \tan^2 \beta} - \frac{m_Z^2}{2}. \quad (3.16c)$$

When $\tan \beta = \mathcal{O}(10)$, the values of μ , $|m_{H_u}|$ and $|m_S|$ are smaller than M_0 by the factor $\tan \beta$ because $m_{H_d} \simeq A_\lambda \simeq M_0$. Thus, the values of μ and $|m_{H_u}|$ could be of $\mathcal{O}(100)$ GeV while the other masses of the superpartners are of $\mathcal{O}(M_0) = \mathcal{O}(1)$ TeV. Then, the fine-tuning problem can be ameliorated. Furthermore, one can see that the first and the second terms in the last equation cancel each other for our choice of c_i . The next leading contributions are of $\mathcal{O}(m_{H_d}^2/\tan^4 \beta)$ or $\mathcal{O}(m_{H_d}^2 \mu/\tan^2 \beta A_\lambda)$. Thus, m_Z^2 is almost determined by $m_{H_u}^2$ alone and insensitive to the value of μ . This means that actually $\tan \beta \approx 3$ is enough to obtain the fine-tuning of $|\partial \ln m_Z^2 / \partial \ln m_{H_u}^2|^{-1} = m_Z^2 / 2m_{H_u}^2 = \mathcal{O}(100)\%$ for $M_0 \approx 1$ TeV. In this case, μ can be as heavy as $\mathcal{O}(400)$ GeV without deteriorating the fine-tuning. The origin of this cancellation is easily understood by examining the doublet mass matrix,

$$\mathcal{L}_M = - (H_d, H_u^*) \mathcal{M}_H \begin{pmatrix} H_d^* \\ H_u \end{pmatrix}, \quad (3.17)$$

where

$$\mathcal{M}_H^2 = \begin{pmatrix} m_{H_d}^2 + \mu^2 & -A_\lambda \mu \\ -A_\lambda \mu & m_{H_u}^2 + \mu^2 \end{pmatrix} \approx \begin{pmatrix} M_0^2 + \mu^2 & -M_0 \mu \\ -M_0 \mu & \mu^2 \end{pmatrix}. \quad (3.18)$$

The modulus mediated contribution M_0 cancels in the determinant of the mass matrix, $\det(\mathcal{M}_H^2) \approx \mu^4$. The heavy mode has mass of $\mathcal{O}(M_0)$, then the mass of the light mode

is suppressed as $\mu^2/M_0 \approx \mu/\tan\beta$ and a flat direction appears along $H_u/H_d \approx M_0/\mu \approx \tan\beta$. This mechanism was previously observed in [11] in the context of the MSSM. In the NMSSM, the relation $m_{H_d} \approx A_\lambda$ is well controlled up to the leading contribution of the modulus mediation, in contrast to the B -term (in place of A_λ) in the MSSM, which is a remnant of the fine-tuned cancellation between the terms of $\mathcal{O}(m_{3/2})$ and subject to uncontrolled corrections.

In the following section, we show numerically the spectrum of our model.

3.3 Spectrum

Here, we study numerically the spectrum of our model. Before showing numerical results, we recall our parameters. In our analysis, the free parameters are λ , κ , $\tan\beta$ and A_κ given at the SUSY scale and M_0 given at M_{GUT} . As the SUSY scale, we choose $M_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \simeq M_0/\sqrt{2}$. Using them, we determine all of the soft SUSY breaking parameters except for $m_{H_u}^2$, m_S^2 and μ , which are determined by using the stationary condition of the Higgs potential. Note that in the following numerical analysis we include corrections from the 1-loop effective potential,

$$V_{1\text{-loop}} = \frac{1}{64\pi^2} Str \left[\mathcal{M}^4 \left\{ \ln \left(\frac{\mathcal{M}^2}{M_{SUSY}^2} \right) - \frac{3}{2} \right\} \right], \quad (3.19)$$

in the stationary conditions (3.5), where \mathcal{M} represents the mass matrix of our model and Str denotes the supertrace. In all of the following numerical analysis, we use $A_\kappa = -100$ GeV as a typical value of A_κ . When λ and/or κ are large at the electroweak scale, they blow up below the GUT scale. Thus, we have constraints on large values of λ and κ by requiring that those do not blow up below the GUT scale.

Figure 1 shows the lightest CP-even Higgs mass m_{h1} , soft scalar masses of H_u and S , m_{H_u} and m_S , and μ for $M_0 = 1200$ GeV and $\tan\beta = 3$, in panels (a), (c), (d) and (e), respectively. The panel (b) in the figure shows the coupling squared between the lightest CP-even Higgs and the Z bosons, g_{ZZh1}^2 , as the ratio to the one in the SM, i.e. g_{ZZh1}^2/g_{SM}^2 . The second lightest CP-even Higgs mass m_{h2} is also plotted in panel (f).

In the figure, the red curve corresponds to the values of λ and κ at the electroweak scale, which blow up at the GUT scale. Thus, we exclude the outside of this curve. The gray region around $\kappa = 0$ corresponds to the region where the tachyonic mode appears in the Higgs sector. The yellow region is excluded because the Higgs potential has the false vacua studied in Ref. [20], deeper than the realistic vacuum. From these constraints, the region with small κ/λ is disfavored. The gray region around the red curve indicates the region where the tree level Higgs mass becomes tachyonic and the iterative procedure we employed does not work to estimate the stationary conditions. The quantum corrections (3.19) could lift the tachyonic mass, however, we do not calculate it because the region has already been excluded by the LEP II bound ($m_{h1} > 114.4$ GeV).

The value of μ is around 200–400 GeV, which is consistent with the rough estimation in Eq. (3.16a), i.e. $\mu \sim m_{H_d}^2/(A_\lambda \tan\beta) \sim M_0/\tan\beta$. Obviously the expansion in Eq. (3.16a) becomes worse for $\kappa/\lambda \gtrsim \tan\beta$, while the expansion holds well for $\kappa/\lambda \lesssim 0.3$ where the

value of $|m_{H_u}|$ is around 100–200 GeV. In this region the fine-tuning of the parameters for the electroweak symmetry breaking is of $\mathcal{O}(10)\%$, even though most of the superpartners have heavy masses of $\mathcal{O}(M_0)$. A large value of $\mu \simeq 400$ GeV potentially degrades the fine-tuning, however the cancellation renders μ irrelevant to m_Z at the leading order of the expansion by $1/\tan\beta$.

The value of $|m_S|$ is roughly estimated as $|m_S| \sim (\sqrt{2}\kappa/\lambda)\mu$ in Eq. (3.16b). Thus, it is found that $|m_S| \approx 400$ GeV for $\kappa \approx \lambda$ and $|m_S|$ increases (decreases) as κ/λ increases (decreases). The value of $|m_S|$ is expected to be suppressed in the TeV scale mirage scenario. However, a large value of κ/λ leads large $|m_S|$ through the stationary condition like $|m_S| \sim 500$ GeV. Such a large value would not be realized in our TeV scale mirage mediation scenario, because m_S^2 must be suppressed compared with M_0^2 . Thus, the region with large κ/λ and $|m_S| > \mathcal{O}(M_0/\sqrt{8\pi^2})$ is disfavored. In the panel (d), the region with $|m_S| > M_0/\sqrt{8\pi^2}$ is filled in pink. It is interesting that the region favored by the TeV scale mirage mediation exactly corresponds to the region where the fine-tuning is ameliorated. Note that the condition (2.10) also holds well for this region.

In Fig. 1.(a), the lightest CP-even Higgs boson, which dominantly consists of the doublet scalar here, has a mass m_{h_1} around 80–125 GeV. We estimated m_{h_1} using NMHDECAY in the NMSSMTools package [21]. We calculated the minimum of the effective potential renormalized at M_{SUSY} by the iteration starting from the tree-level minimum. Then we used the resultant μ as an input of the NMSSMTools. The pole mass $m_t = 172.9$ GeV was used in our calculation. The qualitative behavior of the mass of the SM-like Higgs boson is given by

$$m_{\text{SM}}^2 \simeq m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \frac{\lambda^2}{\kappa^2} v^2 (\lambda - \kappa \sin 2\beta)^2 + \frac{3m_t^4}{4\pi^2 v^2} \left(\ln \left(\frac{m_t^2}{m_t^2} \right) + \frac{A_t^2}{m_t^2} \left(1 - \frac{A_t^2}{12m_t^2} \right) \right), \quad (3.20)$$

for $\kappa s \gg |A_\kappa|, |A_\lambda|$ in [14]. The first term and the fourth term are the tree-level contribution and the radiative correction in the MSSM, respectively. The second term comes from the new quartic Higgs couplings in the NMSSM, and the third term comes from the mixing of the doublet scalars with the singlet. The third term is always negative because the mixing reduces the lightest eigenvalue of the mass matrix. Although the above approximation does not apply in the figure, the behavior of the higgs mass can be understood in terms of the mixing. The effect of the mixing undermines that of the additional quartic coupling $\lambda^2 \sin^2 2\beta$ and suppresses the Higgs mass except for the narrow region $4 \lesssim \lambda/\kappa \lesssim 8$. The LHC observation, $m_{h_1} \approx 125$ GeV, is satisfied only around $(\lambda, \kappa) = (0.7, 0.11)$ where the mixing vanishes. Such a region may be realized as a quasi-infrared fixed point if λ has a strong dynamics origin at the GUT scale, while κ is suppressed due to the approximate Peccei-Quinn symmetry. It is important to stress again that this region is favored by the TeV scale mirage mediation and the fine-tuning. The mixing of the doublet and the singlet Higgs scalars in the Lagrangian is obtained by rotating the doublet Higgs by

$$\beta (= \tan^{-1} v_u/v_d)$$

$$\Delta\mathcal{L} = -2v\lambda [\mu - (A_\lambda + 2\kappa s) \cos\beta \sin\beta] h\Delta S_r, \quad (3.21)$$

where $h = \text{Re}(H_d) \cos\beta + \text{Re}(H_u) \sin\beta - v$ and $\Delta S_r = \text{Re}(S) - s$ denote dynamical degree of freedom of the corresponding Higgs fields. Since our TeV scale mirage mediation scenario leads $\mu \approx M_0/\tan\beta$ and $A_\lambda \approx M_0$, the above term is approximated as,

$$\Delta\mathcal{L} = 4v \frac{\mu}{\tan\beta} \left[\frac{\kappa}{\lambda} + O\left(\frac{1}{\tan^2\beta}\right) \right] h\Delta S_r. \quad (3.22)$$

Thus the mixing is automatically suppressed by $(\kappa/\lambda)/\tan\beta$ in our scenario.

The coupling of the lightest CP-even Higgs boson to the Z boson, g_{ZZh_1} , is almost the same as the one in the standard model in most of the parameter space as expected. We show the ratio $g_{ZZh_1}^2/g_{\text{SM}}^2$ in the figure, where g_{SM} denotes the Higgs coupling to the Z boson in the standard model. The mixing between the doublet and the singlet is minimized around $m_{h_1} \approx 125$ GeV, where g_{ZZh_1} also approaches to its SM value.

Table 1 shows examples of spectra and $g_{ZZh_1}^2/g_{\text{SM}}^2$ for $(\lambda, \kappa) = (0.10, 0.40)$, $(0.40, 0.10)$ and $(0.70, 0.11)$. In the table, m_{h_i} for $i = 1, 2, 3$ and m_{a_i} for $i = 1, 2$ denote three CP-even Higgs masses and two CP-odd Higgs masses, respectively. Also, $m_{\tilde{t}_{1,2}}$ denote two eigenvalues of stop masses. Other squark and slepton masses depend on their values of c_i , and they are of $\mathcal{O}(M_0)$ unless $c_i = 0$. The second lightest CP-even Higgs boson h_2 is lighter for $(\lambda, \kappa) = (0.40, 0.10)$ and $(0.70, 0.11)$ than the one for $(\lambda, \kappa) = (0.10, 0.40)$. Its dominant component is the singlet Higgs boson S . Its mass decreases as κ/λ decreases. Then, the above behavior of m_{h_2} occurs. On the other hand, the dominant component of the heaviest CP-even Higgs boson h_3 is the down-type Higgs boson H_d . Its mass is heavy and almost equal to M_0 , independent of λ and κ . By the same reason, the mass of the heavier CP-odd Higgs boson a_2 is almost the same as m_{h_3} as well as M_0 . The lightest CP-odd Higgs boson a_1 is lighter for small κ due to the approximate Peccei-Quinn symmetry. Also, three gaugino masses are almost the same as M_0 . It might be challenging but interesting subject to observe these light extra-Higgs bosons through the small mixing with the doublets in LHC and ILC.

Figure 2 shows the same as Fig. 1 except for $M_0 = 1500$ GeV. Most of mass parameters become larger than those for $M_0 = 1200$ GeV. The lightest CP-even Higgs mass becomes heavy due to the heavier stop and the portion of the region with $m_{h_1} \approx 125$ GeV increases. Table 2 is the same as Table 1 except $M_0 = 1500$ GeV. Obviously, all of masses become heavier than those in Table 1. The behavior of m_{h_2} , m_{h_3} and m_{a_2} is the same as the one in Table 1. For completeness, we also plot the figure for $M_0 = 1700$ GeV in Fig. 3 and list the spectra and the coupling for $M_0 = 1700$ GeV in Table. 3.

Figure 4 shows the same as Fig. 1 except for $\tan\beta = 5$. The approximation in Eq. (3.16b) and the cancellation work well for $\kappa/\lambda \lesssim 5$. The up-type Higgs mass $|m_{H_u}|$ is around 100 – 250 GeV in most of the parameter space, while the region, $\kappa/\lambda \lesssim 0.6$, satisfies $|m_S| \lesssim 100$ GeV and is favored by the TeV scale mirage mediation. In this region, the singlet becomes lighter than the doublet and $g_{ZZh_1}^2/g_{\text{SM}}^2$ can decrease to $O(10)\%$ near

(λ, κ)	(0.10, 0.40)	(0.40, 0.10)	(0.70, 0.11)
m_{h1}	105 GeV	107 GeV	126 GeV
m_{h2}	1261 GeV	182 GeV	138 GeV
m_{h3}	1700 GeV	1312 GeV	1320 GeV
m_{a1}	512 GeV	181 GeV	158 GeV
m_{a2}	1260 GeV	1311 GeV	1321 GeV
$g_{ZZh_1}^2/g_{\text{SM}}^2$	1.00	0.95	0.94
m_{Hd}^2	$1.39 \times 10^6 \text{GeV}^2$	$1.39 \times 10^6 \text{GeV}^2$	$1.40 \times 10^6 \text{GeV}^2$
m_{Hu}^2	$1.29 \times 10^5 \text{GeV}^2$	$5.58 \times 10^4 \text{GeV}^2$	$3.06 \times 10^4 \text{GeV}^2$
m_S^2	$-1.42 \times 10^6 \text{GeV}^2$	$-1.02 \times 10^4 \text{GeV}^2$	$-4.93 \times 10^3 \text{GeV}^2$
$m_{\tilde{t}_1}$	823 GeV	823 GeV	823 GeV
$m_{\tilde{t}_2}$	849 GeV	849 GeV	849 GeV
μ	217 GeV	396 GeV	407 GeV

Table 1: Spectra for $(\lambda, \kappa) = (0.10, 0.40)$, $(0.4, 0.10)$ and $(0.7, 0.11)$ with $\tan \beta = 3$ and $M_0 = 1200 \text{GeV}$.

(λ, κ)	(0.10, 0.40)	(0.40, 0.10)	(0.70, 0.11)
m_{h1}	107 GeV	110 GeV	128 GeV
m_{h2}	1574 GeV	231 GeV	172 GeV
m_{h3}	2134 GeV	1637 GeV	1641 GeV
m_{a1}	572 GeV	200 GeV	172 GeV
m_{a2}	1573 GeV	1636 GeV	1645 GeV
$g_{ZZh_1}^2/g_{\text{SM}}^2$	1.00	0.98	0.99
m_{Hd}^2	$2.25 \times 10^6 \text{GeV}^2$	$2.18 \times 10^6 \text{GeV}^2$	$2.18 \times 10^6 \text{GeV}^2$
m_{Hu}^2	$2.03 \times 10^5 \text{GeV}^2$	$5.02 \times 10^4 \text{GeV}^2$	$3.84 \times 10^4 \text{GeV}^2$
m_S^2	$-2.25 \times 10^6 \text{GeV}^2$	$-1.92 \times 10^4 \text{GeV}^2$	$-9.38 \times 10^3 \text{GeV}^2$
$m_{\tilde{t}_1}$	1023 GeV	1023 GeV	1023 GeV
$m_{\tilde{t}_2}$	1055 GeV	1055 GeV	1055
μ	272 GeV	495 GeV	508 GeV

Table 2: Spectra for $(\lambda, \kappa) = (0.10, 0.40)$, $(0.40, 0.10)$ and $(0.70, 0.11)$ with $\tan \beta = 3$ and $M_0 = 1500 \text{GeV}$.

(λ, κ)	(0.10, 0.40)	(0.40, 0.10)	(0.70, 0.11)
m_{h1}	108 GeV	112 GeV	128 GeV
m_{h2}	1782 GeV	264 GeV	196 GeV
m_{h3}	2422 GeV	1853 GeV	1861 GeV
m_{a1}	609 GeV	202 GeV	181 GeV
m_{a2}	1782 GeV	1854 GeV	1861 GeV
$g_{ZZh_1}^2/g_{\text{SM}}^2$	1.00	0.99	1.00
$m_{H_d}^2$	$2.79 \times 10^6 \text{GeV}^2$	$2.80 \times 10^6 \text{GeV}^2$	$2.80 \times 10^6 \text{GeV}^2$
$m_{H_u}^2$	$2.62 \times 10^5 \text{GeV}^2$	$6.56 \times 10^4 \text{GeV}^2$	$5.09 \times 10^4 \text{GeV}^2$
m_S^2	$-2.91 \times 10^6 \text{GeV}^2$	$-2.66 \times 10^4 \text{GeV}^2$	$-1.31 \times 10^4 \text{GeV}^2$
$m_{\tilde{t}_1}$	1157 GeV	1157 GeV	1157 GeV
$m_{\tilde{t}_2}$	1193 GeV	1193 GeV	1193 GeV
μ	308 GeV	560 GeV	575 GeV

Table 3: Spectra for $(\lambda, \kappa) = (0.10, 0.40)$, $(0.40, 0.10)$ and $(0.70, 0.11)$ with $\tan \beta = 3$ and $M_0 = 1700 \text{GeV}$.

the border of the excluded region by the false vacuum. In such a region the lightest CP-even Higgs boson ($m_{h_1} \simeq 80 - 90 \text{ GeV}$) could escape the LEP II bound and the second lightest CP-even Higgs boson $m_{h_2} \approx 125 \text{ GeV}$ gives the signal observed in LHC [22]. Note that the small mixing with the singlet scalar enhances the second lightest Higgs mass in contrast to the lightest Higgs case and makes it easier to obtain the LHC value. The production cross section of h_2 (mainly composed of H_u) through the gluon fusion or the vector boson fusion is reduced by $O(10)\%$ relative to the SM due to the mixing with the singlet. The branching ratio $BR(h_2 \rightarrow ZZ, WW, \gamma\gamma)$ is also reduced. This is because the width $\Gamma(h_2 \rightarrow b\bar{b})$ does not change due to the suppressed mixing between the heavy H_d and the singlet, while the widths ($h_2 \rightarrow ZZ, WW, \gamma\gamma$) are reduced due to the mixing between H_u and the singlet. Such a reduction will be confirmed or excluded by the future measurement of the Higgs coupling in LHC and ILC. Table 4 shows the examples of the mass spectra and $g_{ZZh_1}^2/g_{\text{SM}}^2$ for $(\lambda, \kappa) = (0.10, 0.40)$, $(0.40, 0.10)$ and $(0.70, 0.11)$.

When we increase $\tan \beta$, $|\mu|$ decreases as expected from the rough estimation, $|\mu| \sim M_0/\tan \beta$ in Eq. (3.16a). For example, the value $\tan \beta = 10$ leads $\mu = 100 \text{ GeV}$ or less for $M_0 \simeq 1 \text{ TeV}$, which is excluded by the chargino mass bound by LEP II. The value of $|m_S|$ also decreases as $\tan \beta$ increases. In opposite view, this means that tuning of $|m_S|$ (or $\delta c_i^{(\text{loop})}$) increases to obtain large $\tan \beta$ for fixed (λ, κ) and small $\tan \beta$ is favored in our scenario.

Finally we comment on the fermionic sector. The singlino mass is estimated as $2\kappa\mu/\lambda$. Since we have μ around $200 - 500 \text{ GeV}$, the singlino can also be light. Note that the region with small κ/λ is excluded by the appearance of the tachyonic modes and/or the false vacua. Thus, we can not lead the singlino much lighter than the higgsino. Also, large κ/λ leads large $|m_S|$, which could not be derived in our TeV scale mirage scenario. Thus,

(λ, κ)	(0.10, 0.40)	(0.40, 0.10)	(0.70, 0.11)
m_{h1}	113 GeV	79 GeV	65 GeV
m_{h2}	1136 GeV	126 GeV	130 GeV
m_{h3}	1209 GeV	1222 GeV	1228 GeV
m_{a1}	420 GeV	138 GeV	121 GeV
m_{a2}	1205 GeV	1221 GeV	1227 GeV
$g_{ZZh_1}^2/g_{\text{SM}}^2$	1.00	0.24	0.13
m_{Hd}^2	$1.39 \times 10^6 \text{GeV}^2$	$1.39 \times 10^6 \text{GeV}^2$	$1.39 \times 10^6 \text{GeV}^2$
m_{Hu}^2	$4.91 \times 10^4 \text{GeV}^2$	$2.00 \times 10^4 \text{GeV}^2$	$1.97 \times 10^4 \text{GeV}^2$
m_S^2	$-6.23 \times 10^5 \text{GeV}^2$	$8.11 \times 10^2 \text{GeV}^2$	$4.85 \times 10^3 \text{GeV}^2$
$m_{\tilde{t}_1}$	822 GeV	822 GeV	822 GeV
$m_{\tilde{t}_2}$	847 GeV	847 GeV	847 GeV
μ	146 GeV	228 GeV	232 GeV

Table 4: Spectra for $(\lambda, \kappa) = (0.10, 0.40)$, $(0.40, 0.10)$ and $(0.70, 0.11)$ with $\tan \beta = 5$ and $M_0 = 1200 \text{GeV}$.

singlino much heavier than the higgsino is disfavored. Thus, both masses of the higgsino and singlino would be of the same order. Since all of gaugino masses as well as squark and slepton masses are much heavier, the lightest superparticle would be a linear combination between the higgsino and singlino, depending on κ/λ . The gravitino is quite heavy such as $m_{3/2} \sim 4\pi^2 M_0$, as already known in the mirage mediation mechanism [4, 5, 6].

The lightest neutralino in this model behaves like the linear combination between the higgsino and bino in the MSSM and could reproduce the observed abundance of the cold dark matter assuming the thermal relic saturates it [23]. While the extra Higgs bosons play an important role in the direct detection of them, which could be significantly different from the results in the MSSM [24]. The detailed study of the phenomenology of the TeV scale mirage mediation in the NMSSM is interesting for its distinct mass spectrum, however, beyond the scope of this work [25].

4 Conclusion

We have studied the NMSSM with the TeV scale mirage mediation. The region with large κ/λ requires a large value of $|m_S|$ to satisfy the stationary conditions of the Higgs potential. Such a large value could not be realized in our TeV scale mirage scenario and therefore such parameter region is disfavored. In the favored region, it is found that we can realize $|m_{H_u}| \sim 200 \text{ GeV}$, while other masses are heavy such as 1 TeV. Then, the fine-tuning problem is ameliorated. The cancellation between the effective μ -term and the down-type Higgs soft mass reduces the sensitivity of μ to m_Z and $\mu = \mathcal{O}(500) \text{ GeV}$ is possible without significantly deteriorating the fine-tuning. The mixing between the light doublet and the singlet is suppressed by $(\kappa/\lambda) \tan^{-1} \beta$. For small $\tan \beta$ the lightest

CP-even Higgs is mainly the doublet and its mass reaches 125 GeV without suppression by the mixing. When we increase M_0 , the Higgs mass m_{h_1} slowly increases, and also the value of $|\mu|$ increases. However, the required fine-tuning is still mild e.g. for $M_0 = 1700$ GeV.

The coupling between the lightest CP-even Higgs boson and the Z boson is almost the same as one in the standard model for small $\tan\beta$, however, can decrease to $O(10)\%$ for moderate $\tan\beta$ where the lightest CP-even Higgs is mainly singlet. If the lightest CP-even Higgs escapes the LEP II bound, the second lightest CP-even Higgs boson could be the boson observed in LHC. The mass of the second lightest CP-even Higgs boson depends on κ and λ , and it can be light in the parameter region favored in our scenario. The heaviest CP-even Higgs mass as well as the heaviest CP-odd and charged ones is of $O(M_0)$. The lightest CP-odd Higgs can also be light. Thus, the Higgs sector has a rich structure.

In our scenario, the higgsino is light compared with three gauginos. In addition, the singlino is also light. Both masses of the higgsino and the singlino are of the same order. Then, the lightest superparticle is a linear combination between the higgsino and singlino. Such a neutralino sector and Higgs sector would lead to several phenomenologically interesting aspects.

Note added

While this work was being completed, we received Ref. [26], which also considered relevant aspects.

Acknowledgments

T. K. is supported in part by a Grant-in-Aid for Scientific Research No. 20540266 and the Grant-in-Aid for the Global COE Program “The Next Generation of Physics, Spun from Universality and Emergence” from the Ministry of Education, Culture, Sports, Science and Technology of Japan. K. O. is supported in part by a Grant-in-Aid for Scientific Research No. 21740155 and No. 18071001 from the MEXT of Japan. T. S. is a Yukawa Fellow and his work is partially supported by the Yukawa Memorial Foundation, a Grant-in-Aid for Young Scientists (B) No. 23740190 and the Sasakawa Scientific Research Grant from the Japan Science Society. Numerical computation in this work was partly carried out at the Yukawa Institute Computer Facility.

A Soft SUSY breaking terms

Here we give explicitly soft SUSY breaking terms induced by the mirage mediation mechanism in the NMSSM.

In the mirage mediation, the soft parameters at the scale just below M_{GUT} are given

by

$$\begin{aligned}
M_a(M_{GUT}) &= M_0 + \frac{m_{3/2}}{8\pi^2} b_a g_a^2, \\
A_{ijk}(M_{GUT}) &= (c_i + c_j + c_k) M_0 - (\gamma_i + \gamma_j + \gamma_k) \frac{m_{3/2}}{8\pi^2}, \\
m_i^2(M_{GUT}) &= c_i M_0^2 - \dot{\gamma}_i \left(\frac{m_{3/2}}{8\pi^2} \right)^2 - \frac{m_{3/2}}{8\pi^2} M_0 \theta_i,
\end{aligned} \tag{A.1}$$

where

$$\begin{aligned}
b_a &= -3\text{tr}(T_a^2(\text{Adj})) + \sum_i \text{tr}(T_a^2(\phi^i)), \\
\gamma_i &= 2 \sum_a g_a^2 C_2^a(\phi^i) - \frac{1}{2} \sum_{jk} |y_{ijk}|^2, \\
\theta_i &= 4 \sum_a g_a^2 C_2^a(\phi^i) - \sum_{jk} a_{ijk} |y_{ijk}|^2, \\
\dot{\gamma}_i &= 8\pi^2 \frac{d\gamma_i}{d \ln \mu_R}.
\end{aligned} \tag{A.2}$$

Here, $T_a^2(\text{Adj})$ and $T_a^2(\phi^i)$ denote Dynkin indices of the adjoint representation and the representation of matter fields ϕ^i . We have assumed $\omega_{ij} = \sum_{kl} y_{ijk} y_{jkl}^*$ to be diagonal.

Within the framework of the NMSSM, the β -function coefficients, anomalous dimensions and other coefficients in the above equations are obtained as

$$\begin{aligned}
b_3 &= -3, \quad b_2 = 1, \quad b_1 = 11, \\
\gamma_{H_u} &= \frac{3}{2} g_2^2 + \frac{1}{2} g_1^2 - 3y_t^2 - \lambda^2, \\
\gamma_{H_d} &= \frac{3}{2} g_2^2 + \frac{1}{2} g_1^2 - \lambda^2, \\
\gamma_S &= -2\kappa^2 - 2\lambda^2, \\
\gamma_{Q_a} &= \frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{1}{18} g_1^2 - (y_t^2 + y_b^2) \delta_{3a}, \\
\gamma_{U_a} &= \frac{8}{3} g_3^2 + \frac{8}{9} g_1^2 - 2y_t^2 \delta_{3a}, \\
\gamma_{D_a} &= \frac{8}{3} g_3^2 + \frac{2}{9} g_1^2 - 2y_b^2 \delta_{3a}, \\
\gamma_{L_a} &= \frac{3}{2} g_2^2 + \frac{1}{2} g_1^2 - y_\tau^2 \delta_{3a}, \\
\gamma_{E_a} &= 2g_1^2 - 2y_\tau^2 \delta_{3a},
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
\theta_{H_u} &= 3g_2^2 + g_1^2 - 6y_t^2 a_{H_u Q_3 U_3^c} - 2\lambda^2 a_{H_u H_d S}, \\
\theta_{H_d} &= 3g_2^2 + g_1^2 - 6y_b^2 a_{H_d Q_3 D_3^c} - 2y_\tau^2 a_{H_d L_3 E_3^c} - 2\lambda^2 a_{H_u H_d S}, \\
\theta_S &= -2\lambda^2 a_{H_u H_d S} - \kappa^2 a_{SSS}, \\
\theta_{Q_a} &= \frac{16}{3}g_3^2 + 3g_2^2 + \frac{1}{9}g_1^2 - 2(y_t^2 a_{H_u Q_3 U_3^c} + y_b^2 a_{H_d Q_3 D_3^c})\delta_{3a}, \\
\theta_{U_a} &= \frac{16}{3}g_3^2 + \frac{16}{9}g_1^2 - 4y_t^2 a_{H_u Q_3 U_3^c}\delta_{3a}, \\
\theta_{D_a} &= \frac{16}{3}g_3^2 + \frac{4}{9}g_1^2 - 4y_b^2 a_{H_d Q_3 D_3^c}\delta_{3a}, \\
\theta_{L_a} &= 3g_2^2 + g_1^2 - 2y_\tau^2 a_{H_d L_3 E_3^c}\delta_{3a}, \\
\theta_{E_a} &= 4g_1^2 - 4y_\tau^2 a_{H_d L_3 E_3^c}\delta_{3a},
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\dot{\gamma}_{H_u} &= \frac{3}{2}g_2^4 + \frac{11}{2}g_1^4 - 3y_t^2 b_{y_t} - \lambda^2 b_\lambda, \\
\dot{\gamma}_{H_d} &= \frac{3}{2}g_2^4 + \frac{11}{2}g_1^4 - 3y_b^2 b_{y_b} - y_\tau^2 b_{y_\tau} - \lambda^2 b_\lambda, \\
\dot{\gamma}_S &= -2\kappa^2 b_\kappa - 2\lambda^2 b_\lambda, \\
\dot{\gamma}_{Q_a} &= -8g_3^4 + \frac{3}{2}g_2^4 + \frac{11}{18}g_1^4 - (y_t^2 b_{y_t} + y_b^2 b_{y_b})\delta_{3a}, \\
\dot{\gamma}_{U_a} &= -8g_3^4 + \frac{88}{9}g_1^4 - 2y_t^2 b_{y_t}\delta_{3a}, \\
\dot{\gamma}_{D_a} &= -8g_3^4 + \frac{22}{9}g_1^4 - 2y_b^2 b_{y_b}\delta_{3a}, \\
\dot{\gamma}_{L_a} &= \frac{3}{2}g_2^4 + \frac{11}{2}g_1^4 - y_\tau^2 b_{y_\tau}\delta_{3a}, \\
\dot{\gamma}_{E_a} &= 22g_1^4 - 2y_\tau^2 b_{y_\tau}\delta_{3a},
\end{aligned} \tag{A.5}$$

where

$$\begin{aligned}
b_{y_t} &= -\frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2 + 6y_t^2 + y_b^2 + \lambda^2, \\
b_{y_b} &= -\frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{9}g_1^2 + y_t^2 + 6y_b^2 + y_\tau^2 + \lambda^2, \\
b_{y_\tau} &= -3g_2^2 - 3g_1^2 + 3y_b^2 + 4y_\tau^2 + \lambda^2, \\
b_\kappa &= 6\lambda^2 + 6\kappa^2, \\
b_\lambda &= -3g_2^2 - g_1^2 + 3y_t^2 + 3y_b^2 + y_\tau^2 + 4\lambda^2 + 2\kappa^2.
\end{aligned} \tag{A.6}$$

Here, Q_a, U_a, D_a, L_a , and E_a denote left-handed quark, right-handed up-sector quark, right-handed down-sector quark, left-handed lepton, and right-handed lepton fields, respectively, and the index a denotes the generation index. We have included effects due to Yukawa couplings, y_t, y_b , and y_τ , only for the third generations.

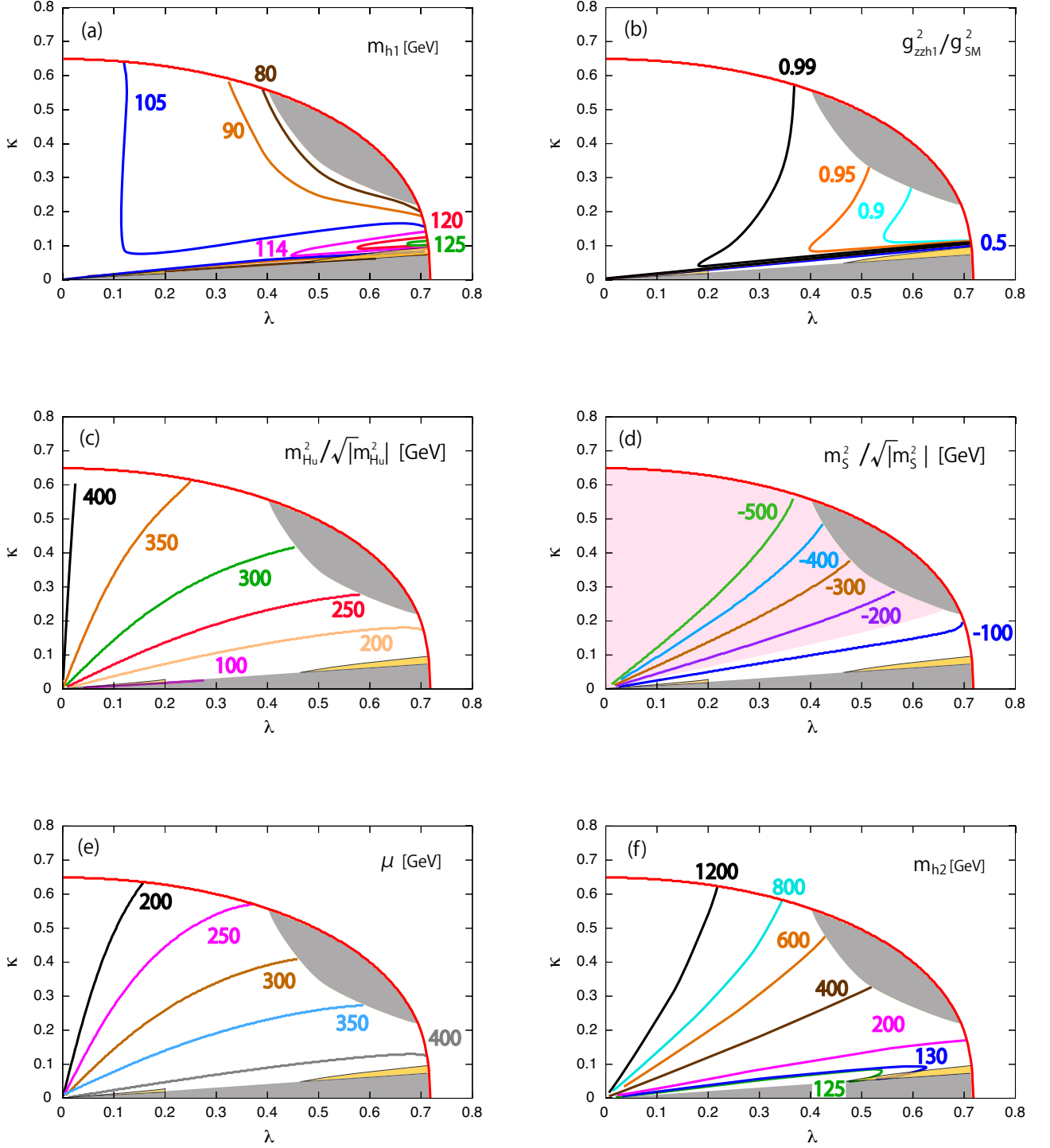


Figure 1: Constraints and SUSY breaking parameters. We use $M_0 = 1200\text{GeV}$, $\alpha = 2$, $c_{Hd} = 1$, $c_{Hu} = 0$, $c_S = 0$, $c_{\tilde{Q}} = 0.5$, $c_{\tilde{t}} = 0.5$ and $\tan\beta = 3$, $A_\kappa = -100\text{GeV}$. For detail of the shaded regions, see the text. The TeV scale mirage mediation disfavors the region $|m_S| \gtrsim M_0/\sqrt{8\pi^2} \sim 100\text{ GeV}$ (pink shaded).

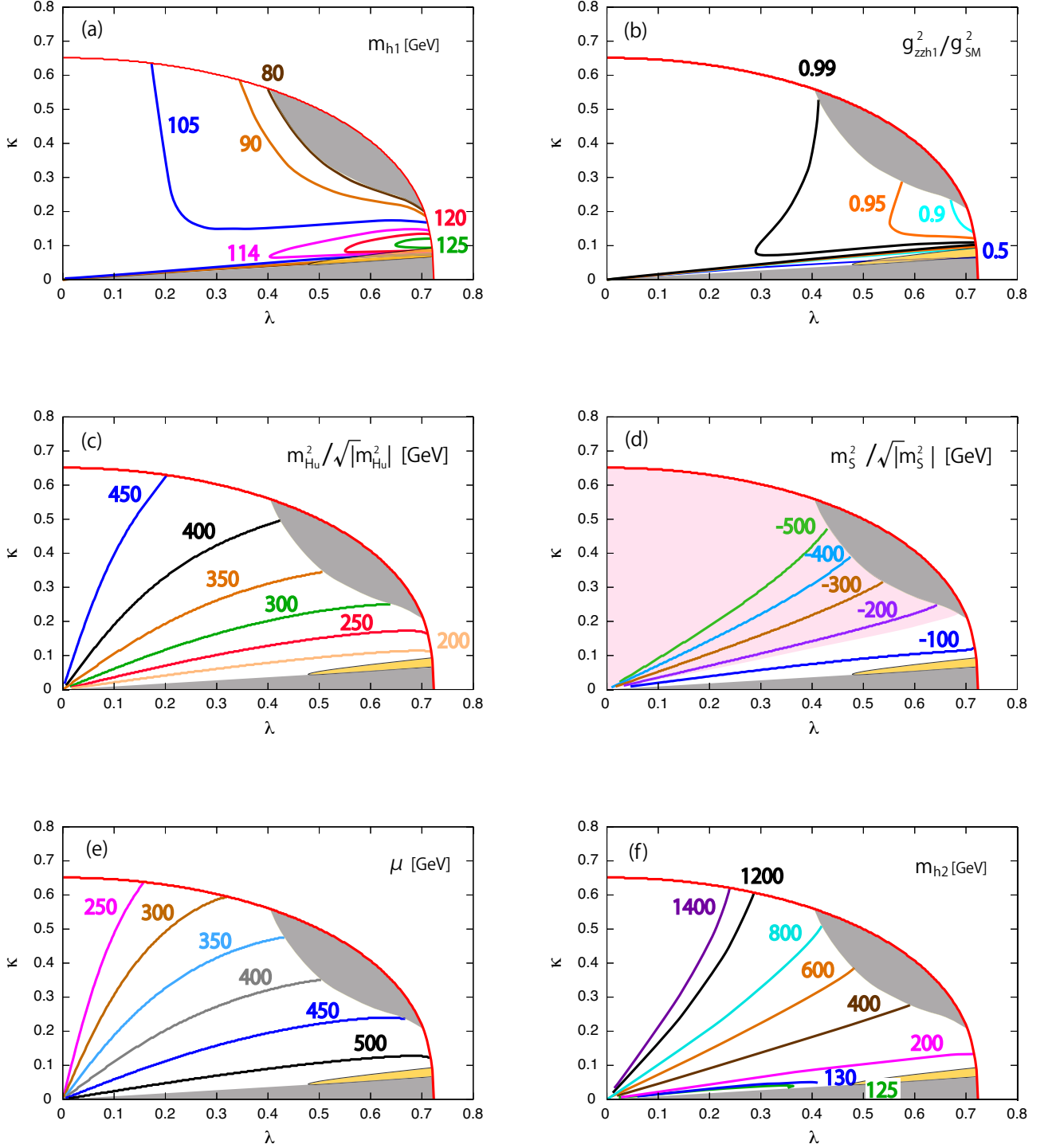


Figure 2: Constraints and SUSY breaking parameters. We use $M_0 = 1500\text{GeV}$, $\alpha = 2$, $c_{Hd} = 1$, $c_{Hu} = 0$, $c_S = 0$, $c_{\tilde{Q}} = 0.5$, $c_{\tilde{t}} = 0.5$ and $\tan\beta = 3$, $A_\kappa = -100\text{GeV}$.

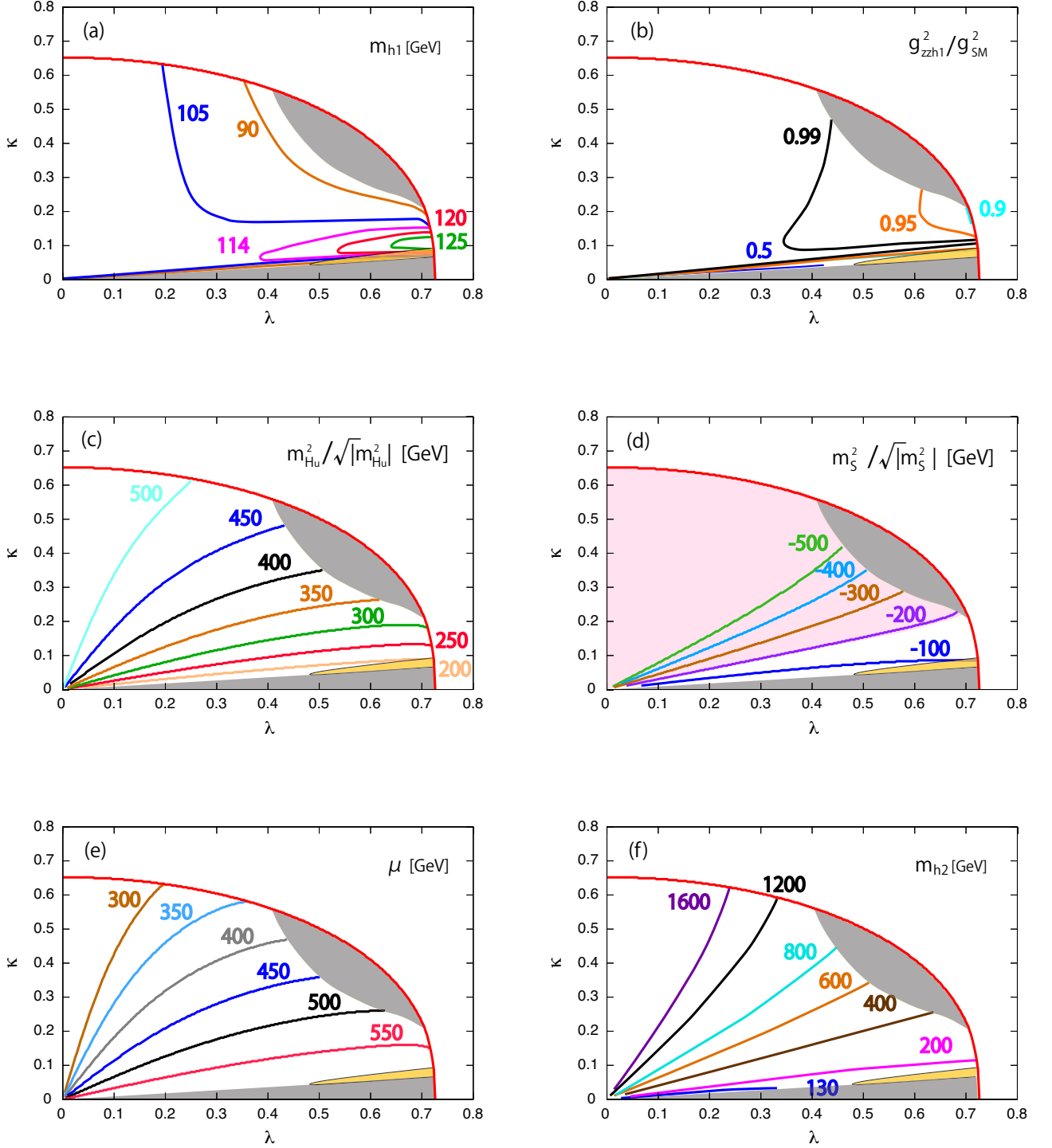


Figure 3: Constraints and SUSY breaking parameters. We use $M_0 = 1700\text{GeV}$, $\alpha = 2$, $c_{Hd} = 1$, $c_{Hu} = 0$, $c_S = 0$, $c_{\tilde{Q}} = 0.5$, $c_{\tilde{t}} = 0.5$ and $\tan\beta = 3$, $A_\kappa = -100\text{GeV}$.

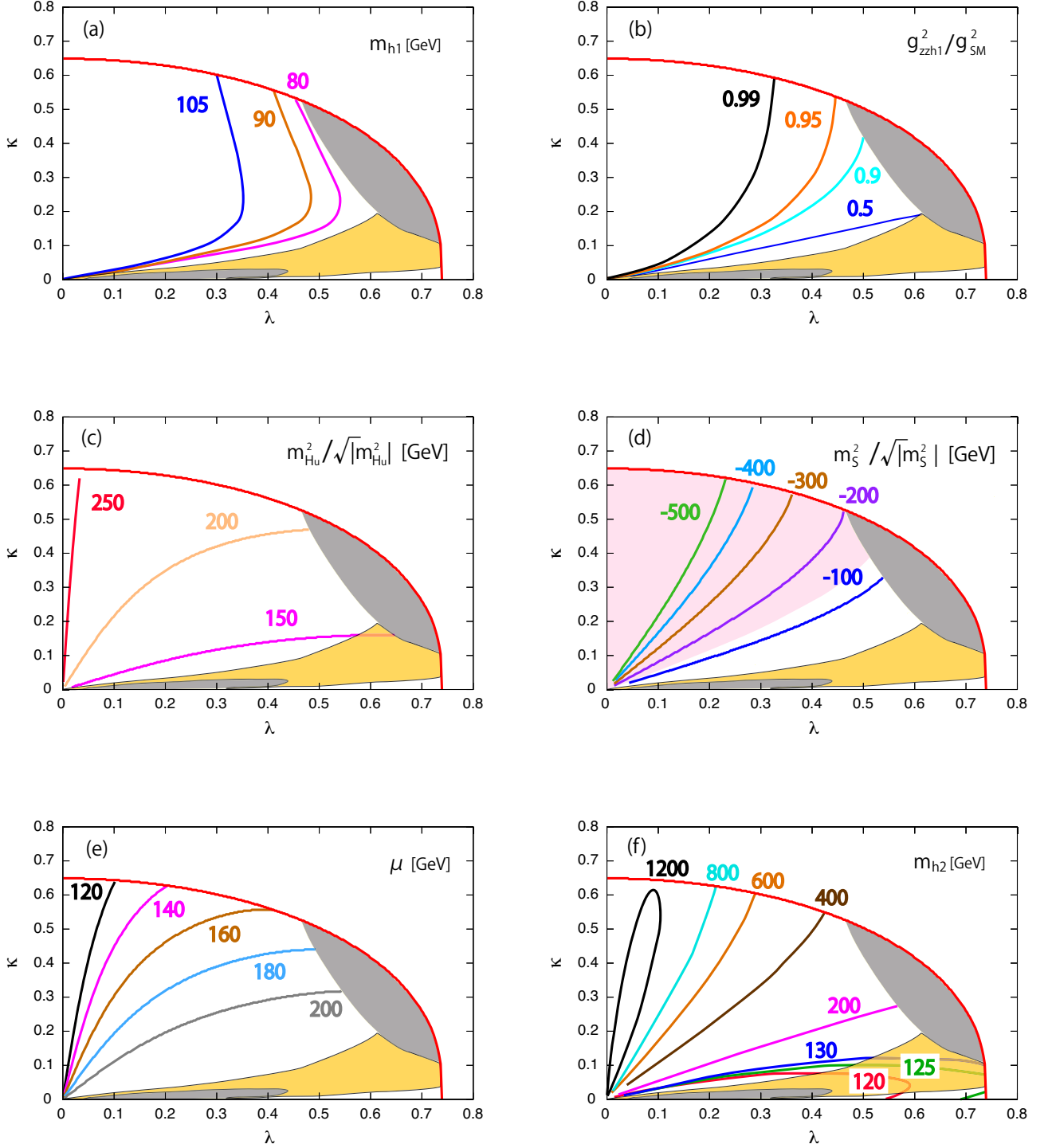


Figure 4: Constraints and SUSY breaking parameters. We use $M_0 = 1200\text{GeV}$, $\alpha = 2$, $c_{Hd} = 1$, $c_{Hu} = 0$, $c_S = 0$, $c_{\tilde{Q}} = 0.5$, $c_{\tilde{t}} = 0.5$ and $\tan \beta = 5$, $A_\kappa = -100\text{GeV}$.

References

- [1] The work in this paper was presented by T. T. at the physical society of Japan annual meeting, Kwansei Gakuin University, 25 March 2012.
- [2] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]].
- [3] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].
- [4] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, JHEP **0411**, 076 (2004) [arXiv:hep-th/0411066]; K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B **718**, 113 (2005) [arXiv:hep-th/0503216].
- [5] K. Choi, K. S. Jeong and K. i. Okumura, JHEP **0509**, 039 (2005) [arXiv:hep-ph/0504037].
- [6] M. Endo, M. Yamaguchi and K. Yoshioka, Phys. Rev. D **72**, 015004 (2005) [arXiv:hep-ph/0504036].
- [7] V. S. Kaplunovsky and J. Louis, Phys. Lett. B **306**, 269 (1993) [arXiv:hep-th/9303040]; A. Brignole, L. E. Ibanez and C. Munoz, Nucl. Phys. B **422**, 125 (1994) [Erratum-ibid. B **436**, 747 (1995)] [arXiv:hep-ph/9308271]; T. Kobayashi, D. Sue-matsu, K. Yamada and Y. Yamagishi, Phys. Lett. B **348**, 402 (1995) [arXiv:hep-ph/9408322]; L. E. Ibanez, C. Munoz and S. Rigolin, Nucl. Phys. B **553**, 43 (1999) [arXiv:hep-ph/9812397].
- [8] L. Randall and R. Sundrum, Nucl. Phys. B **557**, 79 (1999) [arXiv:hep-th/9810155]; G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998) [arXiv:hep-ph/9810442].
- [9] K. Choi, K. S. Jeong, T. Kobayashi and K. i. Okumura, Phys. Lett. B **633**, 355 (2006) [arXiv:hep-ph/0508029].
- [10] R. Kitano and Y. Nomura, Phys. Lett. B **631**, 58 (2005) [arXiv:hep-ph/0509039].
- [11] K. Choi, K. S. Jeong, T. Kobayashi and K. i. Okumura, Phys. Rev. D **75**, 095012 (2007) [arXiv:hep-ph/0612258].
- [12] A. Falkowski, O. Lebedev and Y. Mambrini, JHEP **0511**, 034 (2005) [hep-ph/0507110]; H. Baer, E. -K. Park, X. Tata and T. T. Wang, JHEP **0608**, 041 (2006) [hep-ph/0604253]; JHEP **0706**, 033 (2007) [hep-ph/0703024]; R. Kitano and Y. Nomura, Phys. Rev. D **73**, 095004 (2006) [hep-ph/0602096]; K. Kawagoe and M. M. Nojiri, Phys. Rev. D **74**, 115011 (2006) [hep-ph/0606104]; H. Abe, Y. G. Kim, T. Kobayashi and Y. Shimizu, JHEP **0709**, 107 (2007) [arXiv:0706.4349 [hep-ph]].

- [13] P. Fayet, Nucl. Phys. B **90**, 104 (1975); Phys. Lett. B **64**, 159 (1976); Phys. Lett. B **69**, 489 (1977); Phys. Lett. B **84**, 416 (1979); H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120**, 346 (1983); J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B **222**, 11 (1983); J. P. Derendinger and C. A. Savoy, Nucl. Phys. B **237**, 307 (1984); J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D **39**, 844 (1989); M. Drees, Int. J. Mod. Phys. A **4**, 3635 (1989).
- [14] U. Ellwanger, C. Hugonie and A. M. Teixeira, Phys. Rept. **496**, 1 (2010) [arXiv:0910.1785 [hep-ph]].
- [15] J. E. Kim and H. P. Nilles, Phys. Lett. B **138**, 150 (1984).
- [16] H. Abe, T. Higaki and T. Kobayashi, Phys. Rev. D **73**, 046005 (2006) [hep-th/0511160].
- [17] K. Choi, K. S. Jeong and K. -I. Okumura, JHEP **0807**, 047 (2008) [arXiv:0804.4283 [hep-ph]].
- [18] O. Lebedev, H. P. Nilles and M. Ratz, arXiv:hep-ph/0511320.
- [19] A. Riotto and E. Roulet, Phys. Lett. B **377**, 60 (1996) [hep-ph/9512401]; A. Kusenko, P. Langacker and G. Segre, Phys. Rev. D **54**, 5824 (1996) [hep-ph/9602414]; A. Kusenko and P. Langacker, Phys. Lett. B **391**, 29 (1997) [hep-ph/9608340].
- [20] Y. Kanehata, T. Kobayashi, Y. Konishi, O. Seto and T. Shimomura, Prog. Theor. Phys. **126**, 1051 (2011) [arXiv:1103.5109 [hep-ph]]; T. Kobayashi, T. Shimomura and T. Takahashi, arXiv:1203.4328 [hep-ph].
- [21] U. Ellwanger and C. Hugonie, Comput. Phys. Commun. **175**, 290 (2006) [hep-ph/0508022]; G. Belanger, F. Boudjema, C. Hugonie, A. Pukhov and A. Semenov, JCAP **0509**, 001 (2005) [hep-ph/0505142]; U. Ellwanger, J. F. Gunion and C. Hugonie, JHEP **0502**, 066 (2005) [hep-ph/0406215].
- [22] G. Belanger, U. Ellwanger, J. F. Gunion, Y. Jiang, S. Kraml and J. H. Schwarz, arXiv:1210.1976 [hep-ph]; U. Ellwanger, Eur. Phys. J. C **71**, 1782 (2011) [arXiv:1108.0157 [hep-ph]]; S. F. King, M. Muhlleitner and R. Nevzorov, Nucl. Phys. B **860**, 207 (2012) [arXiv:1201.2671 [hep-ph]]; D. A. Vasquez, G. Belanger, C. Boehm, J. Da Silva, P. Richardson and C. Wymant, Phys. Rev. D **86**, 035023 (2012) [arXiv:1203.3446 [hep-ph]]; U. Ellwanger and C. Hugonie, Adv. High Energy Phys. **2012**, 625389 (2012) [arXiv:1203.5048 [hep-ph]].
- [23] G. Belanger, F. Boudjema, C. Hugonie, A. Pukhov and A. Semenov, JCAP **0509**, 001 (2005) [hep-ph/0505142].
- [24] D. G. Cerdeno, C. Hugonie, D. E. Lopez-Fogliani, C. Munoz and A. M. Teixeira, JHEP **0412**, 048 (2004) [hep-ph/0408102].

- [25] T. Kobayashi, H. Makino, K. i. Okumura, T. Shimomura and T. Takahashi in progress.
- [26] M. Asano and T. Higaki, arXiv:1204.0508 [hep-ph].